

## Topic B: Solving linear equations and rearranging formulae

 Bridging  
to Ch1.4

This topic recaps the **balance** method to solve problems involving linear equations, and both the **elimination** and **substitution** methods to solve linear simultaneous equations. You can solve linear equations and inequalities using the **balance** method where the same operation is applied to both sides.

Example 1

Solve the equation  $7x - 5 = 3x - 2$

$$4x - 5 = -2$$

$$4x = 3$$

$$x = \frac{3}{4}$$

Divide both sides of the equation by 4

Subtract  $3x$  from both sides of the equation.

Add 5 to both sides of the equation.

Solve the equation  $3x + 8 = 5x - 6$

Try It 1

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Example 2

Solve the inequality  $5(x - 2) \leq 2x + 1$

$$5x - 10 \leq 2x + 1$$

$$3x - 10 \leq 1$$

$$3x \leq 11$$

$$x \leq \frac{11}{3}$$

First expand the brackets.

Subtract  $2x$  from both sides.

Add 10 to both sides.

Divide both sides by 3

Solve the inequality  $7x - 4 > x + 8$

Try It 2

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When solving inequalities, remember that multiplying or dividing by a negative number will reverse the inequality sign. For example,  $5 > 3$  but  $-5 < -3$

Equations and formulae can be rearranged using the same method as for solving equations.

Example 3

Rearrange  $Ax - 3 = \frac{x+B}{2}$  to make  $x$  the subject.

$$2Ax - 6 = x + B$$

$$2Ax - 6 - x = B$$

$$2Ax - x = B + 6$$

$$x(2A - 1) = B + 6$$

$$x = \frac{B + 6}{2A - 1}$$

Multiply both sides by 2

Subtract  $x$  from both sides.

Add 6 to both sides.

Divide both sides by  $(2A - 1)$  to make  $x$  the subject.

Factorise the side involving  $x$

Rearrange  $3(x + A) = Bx + 1$  to make  $x$  the subject.

Try It 3

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You can solve linear simultaneous equations using the **elimination** method, as shown in Example 4. The solutions to simultaneous equations give the point of intersection between the lines represented by the two equations.

# Example 4

Solve the simultaneous equations  $5x - 4y = 17$ ,  $3x + 8y = 5$

$15x + 40y = 25$  (1)   
 $15x - 12y = 51$  (2)   
 $(1) - (2): 52y = -26$    
 $y = -\frac{1}{2}$    
 $5x - 4\left(-\frac{1}{2}\right) = 17$    
 $5x + 2 = 17$    
 $5x = 15$    
 $x = 3$

- Multiply the second equation by 5
- Multiply the first equation by 3
- Subtract equation (2) from equation (1) to eliminate  $x$
- Substitute  $y = -\frac{1}{2}$  into one of the original equations.

Solve this equation to find the value of  $x$

Try It 4

Solve the simultaneous equations  $2x + 5y = 1$ ,  $3x - 2y = -27$

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Calculator



### Try it on your calculator

You can use a calculator to solve linear simultaneous equations.

$anX+bnY=Cn$		
1	a	b
2	3	-1
	1	2
$\left[ \begin{array}{cc} 1 & 3 \\ 2 & -1 \end{array} \right] \begin{array}{c} -13 \\ 2 \end{array}$		
[SOLV] [DEL] [CLR] [EDIT]		
2		

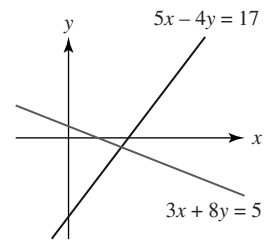
$anX+bnY=Cn$		
x	4	
y	-1	
[REPT]		
4		

### Activity

Find out how to solve the simultaneous equations  $3x - y = 13$  and  $x + 2y = 2$  on *your* calculator.

The example shows you that the lines  $5x - 4y = 17$  and  $3x + 8y = 5$  intersect at the point  $\left(3, -\frac{1}{2}\right)$

If you are given the equation of two lines where  $y$  is the subject then the easiest way to solve these simultaneously is to use the **substitution** method as shown in the next example.



**Example 5**

Find the point of intersection between the lines with equations  $y = 2x + 5$  and  $y = 7 - 3x$

$$2x + 5 = 7 - 3x$$

$$5x + 5 = 7$$

$$5x = 2$$

$$x = 0.4$$

$$y = 2(0.4) + 5$$

$$= 5.8$$

So the lines intersect at the point  $(0.4, 5.8)$

Substitute  $2x + 5$  for  $y$  in the second equation.

Solve to find the value of  $x$

Substitute  $x = 0.4$  into either of the original equations to find the  $y$ -coordinate.

Find the point of intersection between the lines  $y = 3x + 4$  and  $y = 6x - 2$

**Try It 5**

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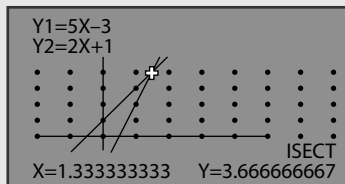
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**Calculator**



**Try it on your calculator**

You can use a graphics calculator to find the point of intersection of two lines.



**Activity**

Find the point of intersection of the lines  $y = 5x - 3$  and  $y = 2x + 1$  on your graphics calculator.





1 Solve each of these linear equations.

**a**  $3(2x+9)=7$

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**b**  $7-3x=12$

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**c**  $\frac{x+4}{5}=7$

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**d**  $2x+7=5x-6$

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**e**  $8x-3=2(3x+1)$

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**f**  $\frac{2x+9}{12} = x-1$

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**g**  $2(3x-7) = 4x$

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**h**  $7-2x = 3(4-5x)$

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**2** Solve each of these linear inequalities.

**a**  $\frac{x}{2} + 7 \geq 5$

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**b**  $3-4x < 15$

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**c**  $5(x-1) > 12+x$

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**d**  $\frac{x+1}{3} > 2$

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**e**  $8x-1 \leq 2x-5$

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**f**  $3(x+1) \geq \frac{x-3}{2}$

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**g**  $3(2x-5) < 1-x$

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**h**  $x - (3 + 2x) \geq 2(x + 1)$

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**3** Rearrange each of these formulae to make  $x$  the subject.

**a**  $2x + 5 = 3A - 1$

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**b**  $x + u = vx + 3$

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**c**  $\frac{3x - 1}{k} = 2x$

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**d**  $5(x-3m)=2nx-4$

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**e**  $(1-3x)^2=t$

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**f**  $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

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**g**  $\frac{1}{x^2+k}-6=4$

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**h**  $\sqrt{x+A} = 2B$

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**4** Use algebra to solve each of these pairs of simultaneous equations.

**a**  $5x+12y = -6, \quad x+5y = 4$

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**b**  $7x+5y = 14, \quad 3x+4y = 19$

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**c**  $2x - 5y = 4$ ,  $3x - 8y = 5$

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**d**  $3x - 2y = 2$ ,  $8x + 3y = 4.5$

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**e**  $5x - 2y = 11$ ,  $-2x + 3y = 22$

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**f**  $8x+5y=-0.5, -6x+4y=-3.5$

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**5** Use algebra to find the point of intersection between each pair of lines.

**a**  $y=8-3x, y=2-5x$

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**b**  $y = 7x - 4, y = 3x - 2$

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**c**  $y = 2x + 3, y = 5 - x$

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**d**  $y + 5 = 3x, y = -5x + 7$

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**e**  $y = \frac{1}{2}x + 3, y = 5 - 2x$

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**f**  $y = 3(x + 2), y = 7 - 2x$

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